Teaching Through Problem Solving

IMAPP 2007/2008
PS Heuristics

• Polya (1945)
• Mason, Burton, & Stacey (1985)
Polya (1945)

- How to Solve It
  - Understand the problem
  - Devise a Plan
  - Carry out the Plan
  - Look Back
Mason, Burton, & Stacey (1985)

- Phases
  - Entry
  - Attack
  - Revise

- Processes
  - Specializing
  - Generalizing
## Polya versus Mason et al

### Polya (1945)

<table>
<thead>
<tr>
<th>PS Heuristic</th>
<th>Understand the Problem</th>
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</tr>
</thead>
</table>

### Mason et al. (1985)

<table>
<thead>
<tr>
<th>PS Heuristic (Phases, Processes, States, &amp; Rubric)</th>
<th>Phases (below):</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry</td>
<td>Attack</td>
</tr>
<tr>
<td>Review</td>
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Questions and further discussion

• Problem Solving brings together deep procedural understanding and conceptual understanding – two sides of the coin called mathematical understanding

• Develops habits of mind that fosters thinking mathematically
AN EXISTENCE PROOF
Existence Proof

• Making visible one way to Teach through Problem Solving
  – TIMSS Video Studies: *Japanese 3*
    • *Problems*
    • *Problem Solving*
    • *Teaching through Problem Solving*
TIMSS Video Studies as a Window into Teaching through Problem Solving (J3)

- Reverse engineering the lesson
- Noticing
- Paying attention to taken-for-granted reified practices and unpacking them to develop professional habits of mind

Seeing something once is better than hearing about it one hundred times.

– Confucius
First, the Offering Problem

- It has been one month since Ichiro’s mother entered the hospital. He has decided to give a prayer with his small brother at a local temple every morning so that she will be well soon. There are 18 ten-yen coins in Ichiro’s wallet and just 22 five-yen coins in his younger brother’s wallet. They decided to place one coin from each of them in the offertory box each morning and continue the prayer until either wallet becomes empty. One day they looked into their wallets and found the brother’s amount was bigger than Ichiro’s. How many days since they started prayer?
TIMSS Video Studies as a Window into Teaching through Problem Solving (J3)

• Pre-Viewing Guide
    – Launch, Explore, and Summarize
  – **Mathematics**: What are the mathematical residues?
  – **Teaching**: What was the role of the teacher? What teaching practices were deployed?
  – **Learning**: What were the roles of the students? How were they engaged?
Focus on Mathematics

• Important Mathematics
• Mathematical Language and Symbolization
• Mathematical Storyline (…)
• Mathematical Representations, their sequencing, and their connections
Focus on Teaching

- Pedagogical Flow
- Pedagogical Flexibility
- Tools and use of Tools
- Questions/Questioning
- Response/Responding
Focus on Learning

- Mathematical Residue
- Problematization
- Engagement
- Problem Solving Strategies (...)
- Questions/Questioning
- Mathematical Language
- Tools and use of Tools
Comments

•
Let’s view the video

- TIMSS Video Studies J3
Post-Viewing Think-Phase

• Take 5 minutes to collect your thoughts about what you have just seen
Share Phase

• What? How? Why?
  – Launch, Explore, and Summarize

• **Mathematics**: What are the mathematical residues?

• **Teaching**: What was the role of the teacher? What teaching practices were deployed?

• **Learning**: What were the roles of the students? How were they engaged?
Focus on Mathematics

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Focus on Learning

• Mathematical Residue
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• Mathematical Language
• Tools and use of Tools
Discussing something, seeing it, and then discussing it again is even better.

– Confucius re-examined
What is Teaching Through Problem Solving?

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What is Teaching through Problem Solving?

• Pose a Problem (What is a problem? versus Exercise?)
• Facilitate students’ understanding of the problem
• Allow time for students to explore the messiness of the problem and generate conjectures and understandings
• Provide students time to present, share, and examine solution strategies and solutions
• Summarize the Mathematical Residue
• Encourage students to Reflect on their Learning
  – Part of this can be tasks/assignments to help students reinforce/elaborate/extend their understanding
Teaching Through PS

• Real World or NOT!
• Focus on *Methods*
• Focus on *Connections*
• Focus on *Communication*
• Allow Time
• Build Skill as well as Conceptual Understanding
• Facilitate through *Questioning*
• Focus on the *Mathematics*
How to Support Understanding

• Engage students in solving challenging problems

• *Examine increasingly better solution strategies*

• Provide information for students at just the right times
  – Hiebert and Wearne, 2003, p. 5

• *Allow time for students to reflect*
Comments/Questions?

•
Signposts of Teaching Through PS

• Focus on structure
• Problematization of mathematics for students
  – posing problems that are within students’ reach, allowing them to struggle to find solutions and then examining the methods they have used (Hiebert & Wearne, 2003, p. 6)
  – Tasks must allow students to treat the situation as problematic, as something they need to think about rather than a prescription they need to follow. Secondly, what is problematic about the task should be the mathematics rather than the aspects of the situation. Finally, in order for students to work serious on a task, it must offer students the chance to use skills and knowledge they already possess (Hiebert et al., 1997, p. 18)
• Focus on methods
  – Recall the offering problem from TIMSS Video Studies J3 Classroom Video
Focusing on Methods

• Develops mathematical connections among strategies and develops connections among relationships and representations
  – Progressively formalize the strategies from concrete to the abstract. *Facilitates development of efficient strategies.*

• Enables error analysis.
  – Promotes the use of mistakes as sites for
    • Learning
    • Addressing misconceptions and preconceptions
    • Establishing truth via reason
Focusing on Methods

• Promotes Equity
  – Progressive formalization of strategies
  – Multiplicity of views/voices
    • Makes students’ ideas visible for public consumption and reflection.
  – Focus on reason and not personages/personality

• Maximizes gains from mathematical struggle
  – Promotes active learning and student engagement
  – Promotes metacognition
  – Provides opportunities to engage in meaningful and worthwhile mathematics
The Problem of Teaching  
(Teaching as Problem Solving)

• Can/should tell
  – Conventions [order of operation, etc.]
  – Symbolism and representations [tables, graphs, etc.]
  – Present and re-present at times of need

• Can/should present alternative methods to resolve
  – Those that have not been suggested by students (Hiebert & Wearne, 2003)
  – Tease out additional strategies for PS toolkit
  – Can emphasize/label the efficient method as a capstone to exploration
The Problem of Teaching
(Teaching as Problem Solving)

• Can/should
  – Highlight information/structures in different methods/strategies
  – Highlight connections among methods/strategies
  – Help students focus on what is important mathematically (i.e., what is the mathematical residue? -- see Hiebert et al., 1997)

• Summary
  – Mathematical understanding develops over time via articulated and coherent “mathematical struggle” negotiated in small steps
Proficiency in Teaching Mathematics 1

• Kilpatrick, Swafford, & Findell (2001, p. 10):
  – *Proficiency in teaching mathematics is related to effectiveness: consistently helping students learn worthwhile mathematical content. It also entails versatility: being able to work effectively with a wide variety of students in different environments and across a range of mathematical content.*

• page (10):
  – Despite the common myth that teaching is little more than common sense or that some people are just born teachers, effective teaching practice can be learned.

• page (10):
  – Just as mathematical proficiency itself involves interwoven strands, teaching for mathematical proficiency requires similarly interrelated components: *conceptual understanding* of the core knowledge of mathematics, students, and instructional practices needed for teaching; *procedural fluency* in carrying out basic instructional routines; *strategic competence* in planning effective instruction and solving problems that arise while teaching; ...
Kilpatrick, Swafford, & Findell (2001) cont (3).

• page (10):
  – *adaptive reasoning* in justifying and explaining one’s practices and in reflecting on those practices; and a *productive disposition* towards mathematics, teaching, learning, and the improvement of practice.